FAR BEYOND

MAT122

Inverse Functions

Stony Brook University

Inverse Functions - Intro

Recall: additive inverse $x - 4 = 7$

multiplicative inverse $4x = 7$

opposite operation

inverses are often used to **cancel** and **solve**

Inverse of a Relation

$\{(A,1)\ ,\ (B,12),\ (C,5)\}$ to get the inverse of a relation, reverse the coordinates of the ordered pair

i.e., (x,y) becomes (y,x)

inverse:

$\{(1,A)\ ,\ (12,B),\ (5,C)\}$

note: both original and inverse relations are FUNCTIONS

example:

$\{(P,1)\ ,\ (R,1)\}$ function

$\{(1,P)\ ,\ (1,R)\}$ not a function

Finding the Inverse of Functions

Notation: $f^{-1}(x)$ "f inverse of x" or "inverse of f"

Steps to Find the Inverse of a Function:

1. replace $f(x)$ with y
2. interchange x and y
3. solve for y
4. change y to $f^{-1}(x)$

ex. Find inverse of $f(x) = 7x - 5$.

$$y = 7x - 5$$

$$x = 7y - 5$$

$$+5 \qquad\qquad +5$$

$$\frac{x+5}{7} = \frac{7y}{7} \longrightarrow y = \frac{x+5}{7}$$

$$\boxed{f^{-1}(x) = \frac{x+5}{7}}$$

Follow up Q: find $f^{-1}(9)$

plug in

$$= \frac{9+5}{7} = \frac{14}{7} = \boxed{2}$$

More Finding the Inverse of Functions

ex. Find inverse of $f(x)=\frac{9}{5}x+32.$ (converts Celsius to Fahrenheit)

Steps to Find Inverse:

1. replace $f(x)$ with y
2. interchange x and y
3. solve for y
4. change y to $f^{-1}(x)$

$$y=\frac{9}{5}x+32$$

$$x=\frac{9}{5}y+32$$

$$\frac{5}{9}\left(x-32=\frac{9}{5}y\right)$$

$$\frac{5}{9}\cdot\frac{9}{5}y=y$$

$$\frac{5}{9}(x-32)=y$$

$$\boxed{\frac{5}{9}(x-32)=f^{-1}(x)}$$

(converts Fahrenheit to Celsius)

Finding the Inverse of Function w a Fraction

ex. Find inverse of $f(x)=\dfrac{4x+5}{2x+3}$.

Steps to Find Inverse:

1. replace $f(x)$ with y
2. interchange x and y
3. solve for y
4. change y to $f^{-1}(x)$

$$y=\frac{4x+5}{2x+3}$$

$$x=\frac{4y+5}{2y+3}$$ cross multiply

$$x(2y+3)=4y+5$$ distribute x

$$2xy+3x=4y+5$$ isolate terms w y on LHS:

$$2xy-4y=5-3x$$ factor out y

$$\frac{y(2x-4)}{2x-4}=\frac{5-3x}{2x-4}$$ divide both sides by $2x–4$

$$\boxed{f^{-1}(x)=\frac{5-3x}{2x-4}}$$

$$\begin{array}{rcl} & 2xy+3x=4y+5 & \\ -4y & -3x \quad -4y & -3x \\ \hline & 2xy-4y=5-3x & \end{array}$$

Finding the Inverse of Function - Do

Do: Find inverse of $f(x)=x^3+1$.

$$f^{-1}(x)=\sqrt[3]{x-1}$$

Do: Find inverse of $f(x)=\dfrac{5}{x}+4$.

Do: Find inverse of $f(x)=\dfrac{x+5}{x-4}$.

$$f^{-1}(x)=\frac{5}{x-4}$$

$$f^{-1}(x)=\frac{5+4x}{x-1}$$

Properties of Inverse Functions

<u>Domain and Range</u>

$$\overset{f}{(x, y)} \leftrightarrow \overset{f^{-1}}{(y, x)}$$

domain of f = range of f^{-1}

range of f = domain of f^{-1}

Note: this means the inverse of f^{-1} is f