

**FAR  
BEYOND**

**MAT122**

# **Inverse Functions**



Stony Brook University

# Inverse Functions - Intro

Recall: additive inverse

$$x - 4 = 7$$

multiplicative inverse

$$4x = 7$$

opposite operation

inverses are often used to **cancel** and **solve**

## Inverse of a Relation

$\{(A,1), (B,12), (C,5)\}$  to get the inverse of a relation, reverse the coordinates of the ordered pair  
i.e.,  $(x,y)$  becomes  $(y,x)$

inverse:

$\{(1,A), (12,B), (5,C)\}$

note: both original and inverse relations are FUNCTIONS

example:

$\{(P,1), (R,1)\}$  function

$\{(1,P), (1,R)\}$  not a function

# Finding the Inverse of Functions

Notation:  $f^{-1}(x)$  “ $f$  inverse of  $x$ ” or “inverse of  $f$ ”

## Steps to Find the Inverse of a Function:

1. replace  $f(x)$  with  $y$
2. interchange  $x$  and  $y$
3. solve for  $y$
4. change  $y$  to  $f^{-1}(x)$

ex. Find inverse of  $f(x) = 7x - 5$ .

$$y = 7x - 5$$

$$x = 7y - 5$$

$$\begin{array}{r} +5 \qquad +5 \\ \hline x + 5 = 7y \\ \hline \end{array} \quad \rightarrow \quad y = \frac{x+5}{7}$$

$$f^{-1}(x) = \frac{x+5}{7}$$

Follow up Q: find  $f^{-1}(9)$  plug in  $= \frac{9+5}{7} = \frac{14}{7} = \boxed{2}$

# More Finding the Inverse of Functions

ex. Find inverse of  $f(x) = \frac{9}{5}x + 32$ . (converts Celsius to Fahrenheit)

$$y = \frac{9}{5}x + 32$$

$$x = \frac{9}{5}y + 32$$

$$\frac{5}{9}(x - 32) = \frac{9}{5}y$$

$$\frac{5}{9} \cdot \frac{9}{5}y = y$$

$$\frac{5}{9}(x - 32) = y$$

$$\frac{5}{9}(x - 32) = f^{-1}(x)$$

(converts Fahrenheit to Celsius)

## Steps to Find Inverse:

1. replace  $f(x)$  with  $y$
2. interchange  $x$  and  $y$
3. solve for  $y$
4. change  $y$  to  $f^{-1}(x)$

# Finding the Inverse of Function w a Fraction

ex. Find inverse of  $f(x) = \frac{4x+5}{2x+3}$ .

$$y = \frac{4x+5}{2x+3}$$

$$x = \frac{4y+5}{2y+3}$$

cross multiply

$$x(2y+3) = 4y+5$$

distribute x

$$2xy + 3x = 4y + 5$$

isolate terms w y on LHS:

$$2xy - 4y = 5 - 3x$$

factor out y

$$y \frac{(2x-4)}{2x-4} = \frac{5-3x}{2x-4}$$

divide both sides by  $2x-4$

$$f^{-1}(x) = \frac{5-3x}{2x-4}$$

## Steps to Find Inverse:

1. replace  $f(x)$  with  $y$
2. interchange  $x$  and  $y$
3. solve for  $y$
4. change  $y$  to  $f^{-1}(x)$

$$\begin{array}{r} 2xy + 3x = 4y + 5 \\ -4y \quad -3x \quad -4y \quad -3x \\ \hline 2xy - 4y = 5 - 3x \end{array}$$

# Finding the Inverse of Function - Do

Do: Find inverse of  $f(x) = x^3 + 1$ .

$$f^{-1}(x) = \sqrt[3]{x-1}$$

Do: Find inverse of  $f(x) = \frac{5}{x} + 4$ .

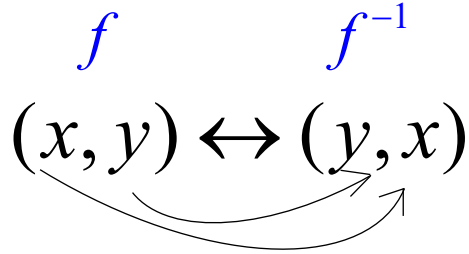
$$f^{-1}(x) = \frac{5}{x-4}$$

Do: Find inverse of  $f(x) = \frac{x+5}{x-4}$ .

$$f^{-1}(x) = \frac{5+4x}{x-1}$$

# Properties of Inverse Functions

## Domain and Range

$$\begin{array}{ccc} f & & f^{-1} \\ (x, y) & \leftrightarrow & (y, x) \end{array}$$


$$\text{domain of } f = \text{range of } f^{-1}$$

$$\text{range of } f = \text{domain of } f^{-1}$$

Note: this means the inverse of  $f^{-1}$  is  $f$